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**A 3-PHASE INDUCTION MACHINE GENERATING SINGLE-PHASE POWER USING  
AN OPTIMIZATION ALGORITHM**

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**ABSTRACT**

This paper presents a performance analysis of a three phase self excited Induction machine as a single phase generator specially suitable for remote and isolated areas of developing countries utilizing local renewable energy sources like wind, small hydro etc, to reduce the burden on main grid and subsequently relaxation for depleting precious conventional energy sources.

A series parallel combination of capacitors has been considered for the self-excitation of the Induction machine being used as generator. In the entire process of power generation from an Induction machine, the magnetization reactance and per unit frequency are the most crucial. A multivariable objective function (absolute value of the complex loop impedance formed using the conditions set in the equivalent ckt of the SEIG system. The unknown variables of the equivalent circuit (magnetizing reactance and generated frequency) of the system are solved by minimizing the objective function considering the bounds of the unknown as external constraint. The constraint optimization problem is converted into a series of unconstrained problem and than Rosenbrock's rotating method of optimization technique is applied to find the minimum value of the objective function. The Sequential Unconstraint Minimization Technique (SUMT) in conjunction with rotating coordinates method of Rosenbrock's has been chosen for calculation of magnetizing reactance and per unit frequency on which the performance characteristics of the Induction generator are derived. The results are reported for a 6KW machine.

**KEYWORDS:** Renewable Energy; Self-Excited Induction generator; Series Parallel Excitation; SUMT; Optimization

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**INTRODUCTION**

In quest for energy, the present world is looking beyond the conventional sources of energy i.e., exploitation of energy from renewable energy sources such as wind, biomass, small hydro etc. Self-excited induction generators [1] are found most suitable for tapping such renewable energy in special context to remote and isolated areas of developing countries. Besides two main inherent problems of poor voltage profile and poor frequency profile, the induction generator have the advantages of low cost, simple and rugged (squirrel cage) construction, easy availability in the market, no maintenance, easy installation, long life, protection against short circuit etc. The village power loads are mostly satisfied by the single-phase supply, which is nothing but the illumination and domestic heating, which is also not so frequency sensitive. Single-phase induction generators are available in literature. But in comparison the three phase machines are more efficient and giving more power output. But problem of phase unbalance is there. One attempt at addressing the voltage regulation weaknesses reported by Murthy is a self-regulated self-excited single-phase induction generator [6], which uses a two-phase squirrel cage induction machine. This generator has two stator windings in quadrature, connected externally to a shunt and series capacitor respectively. By selecting a series capacitor of suitable value along with shunt capacitor, which can reduce the demagnetization with load variation an acceptable voltage regulation can be obtained without further control.

Here in proposed scheme the generator consists of a three phase squirrel cage induction machine star connected with three capacitors and two with one of these capacitors, connected in parallel with the single phase load. With the introduction of this three-phase machine in addition to improved voltage regulation, an additional advantage is also obtained. The power pulsating peculiarity common to single phase induction generators (single phase induction machine used as single phase machine [4-6] under no load does not occur, because the excitation power in this generator is three phase. Moreover, the vibration and noise in the generator caused by this phenomenon can be significantly reduced under load conditions as well. The magnetization reactance and per unit frequency is determined by SUMT in conjunction with rotating coordinates of Rosenbrock's method

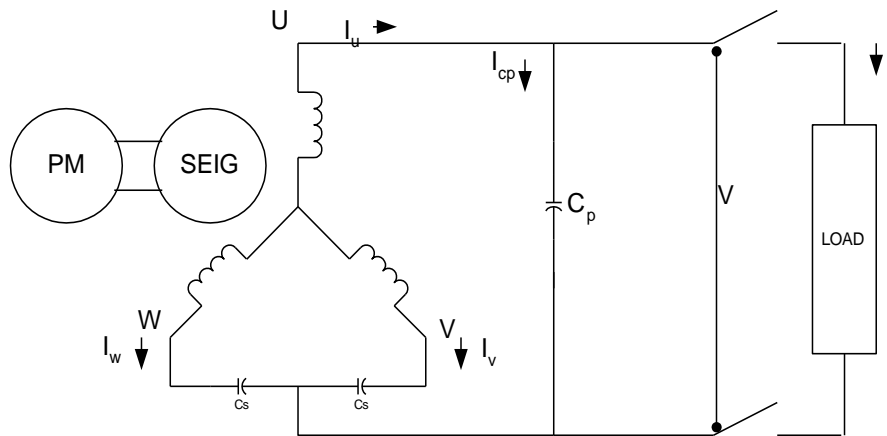


Fig-1. Schematic diagram

### EQUIVALENT CURCUIT

#### Assumptions in Analysis

Following assumptions are considered in this analysis-

- 1- Of all machine parameters, only the magnetizing reactance is affected by magnetic saturation [6,7].
- 2- Leakage reactances of stator and rotor are equal [8].
- 3- Core losses in the machine are neglected.
- 4- Space harmonics in air gap are ignored.

#### Derivation of Equivalent Circuit-

Following symbols have been used in the analysis-

- $V_u, V_v, V_w$  - Stator terminal voltages.
- $V_p, V_n$  - Positive and negative sequence components of stator terminal Voltages.
- $I_u, I_v, I_w, I_{cp}$  - Stator phase currents and parallel capacitor currents.
- $V, I$  - Output voltage and load current.
- $x_{cs}, x_{cp}$  - Capacitive reactances of capacitor  $C_s$  and  $C_p$ .
- $R, X$  - Load resistance and reactance.
- $Z_e$  - Equivalent impedance combined with load and capacitor  $C_p$ .
- $r_1, r_2'$  - Per phase stator and (referred to stator) resistances.
- $x_1, x_2'$  - Per phase stator and (referred to stator) leakage reactances.

- $x_m$  - per phase magnetizing reactance.
- $E_g$  - Air gap voltage.
- $Z_{gp}, Z_{gn}$  - Positive and negative sequence components of the generator Impedance.
- $a$  - Vector operator,  $a = e^{j2\pi/3}$
- $\omega$  - Rotor angular velocity.
- $f_R, \omega_R$  - rated frequency and angular velocity  $\omega_R = 2\pi f_R$ .
- $f, F$  - generation frequency and per unit frequency,  $F = f / f_R$
- $\gamma$  - Per unit rotor speed  $\gamma = \omega / \omega_R$
- $\omega_s$  - Synchronous angular velocity,  $\omega_s = 2\pi f$ .
- $s$  - Slip,  $s = (\omega_s - \omega) / \omega_s = (F - \gamma) / F$ .

(Air gap voltage and all reactances relate to rated frequency)

From Fig-1, this is clear that since generator windings connected in star, the zero sequence components of the stator phase currents do not flow in it. Therefore, expressing the stator phase currents  $I_u, I_v$  and  $I_w$  by symmetrical components, as follows:

$$\left. \begin{aligned} I_u &= I_p + I_n \\ I_v &= a^2 I_p + a I_n \\ I_w &= a I_p + a^2 I_n \end{aligned} \right\} \dots\dots\dots (1)$$

Also from Fig-1 the, terminal voltages  $V_u, V_v, V_w$  can be given as under

$$\left. \begin{aligned} V_u &= Z_e I_u - (-jx_{cs}) I_v \\ V_v &= -jx_{cs} I_v - (-jx_{cs}) I_w \\ V_w &= -jx_{cs} I_w - Z_e I_u \end{aligned} \right\} \dots\dots\dots (2)$$

And also their positive and negative sequence components  $V_p$  and  $V_n$  are represented by the following equations-

$$\left. \begin{aligned} V_p &= 1/3(V_u + aV_v + a^2V_w) \\ &= 1/3\{(Z_e - j2x_{cs})(1 - a^2)I_p + (Z_e + jx_{cs})(1 - a^2)I_n\} \\ V_n &= 1/3(V_u + aV_w + a^2V_v) \\ &= 1/3\{(Z_e + jx_{cs})(1 - a)I_p + (Z_e - j2x_{cs})(1 - a)I_n\} \end{aligned} \right\} \dots\dots\dots (3)$$

If we express  $V_p$  and  $V_n$  in (3) using the positive and negative sequence components of the generator impedance,  $Z_{gp}$  and  $Z_{gn}$ , we obtain

$$\left. \begin{aligned} V_p &= - (1 - a^2)Z_{gp} I_p \\ V_n &= - (1 - a)Z_{gn} I_n \end{aligned} \right\} \dots\dots\dots (4)$$

There fore from (3) and (4) the following impedance matrix related to the the positive and negative sequence components of stator phase currents,  $I_p$  and  $I_n$ , can be obtained

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (1/3)Z_e - j(2/3)x_{cs} + Z_{gp} & 1/3(Z_e + jx_{cs}) \\ (1/3)(Z_e + jx_{cs}) & (1/3)Z_e - j(2/3)x_{cs} + Z_{gn} \end{bmatrix} \begin{bmatrix} I_p \\ I_n \end{bmatrix} \dots\dots\dots(5)$$

In order to include the effect of frequency variation accounted with load change, per unit frequency F and rotor speed y must be included, so an equivalent final diagram of the generator is obtained as shown in Fig-2. Based on this equivalent diagram we will calculate the generators steady-state characteristics.

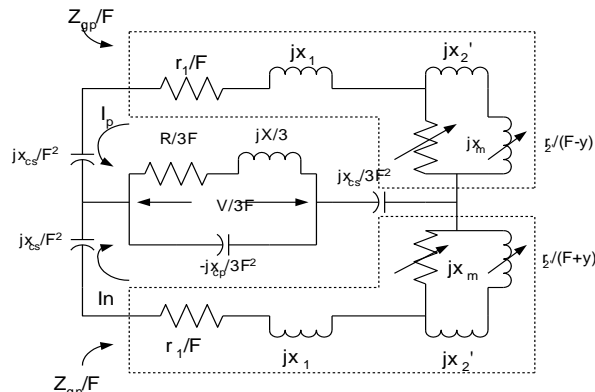


Fig-2 Equivalent circuit diagram of SEIG

**COMPUTATIONAL PROCEDURE OF GENERATOR CHARACTERISTICS**

**Determination of Magnetizing Reactance and per unit Frequency**

In order to determine the generator's characteristics from the equivalent circuit of Fig-2 it is first necessary to determine the magnetizing reactance \$x\_m\$ and per unit frequency \$F\$ for given values of load, capacitors and rotor speed.

From Fig-2, the loop equation for the current \$I\_p\$ can be written

$$ZI_p = 0 \dots\dots\dots (6)$$

Where Z is the total loop impedance seen by \$I\_p\$ and is given by

$$Z = (Z_{gp} / F) - j(x_{cs} / F^2) + ((Z_{gn} / F) - j(x_{cs} / F^2)) * ((Z_e / 3F) + j(x_{cs} / 3F^2)) / ((Z_e / 3F) + j(x_{cs} / 3F^2)) + (Z_{gn} / F) - j(x_{cs} / F^2) \dots\dots\dots(7)$$

Since \$I\_p \neq 0\$ under power generation, \$Z = 0\$ this implies that both the real and imaginary parts of Z must be zero. From this result, two non linear equation with \$x\_m\$ and \$F\$ as unknown variables can be derived. By solving these two equations, \$x\_m\$ and \$F\$ under given condition can be determined. An optimization technique as detailed below has been used to calculate the value of \$x\_m\$ and \$F\$, on which the performance of the machine has been calculated –

**SUMT- Minimization Technique [3]**

A general nonlinear multivariable constrained minimization problem is stated as

$$\text{Find } X(x_1, x_2, x_3, \dots, x_n) \text{ such that}$$

$F(X)$  is minimum subject to  
 and  $x_{li} < x_i \leq x_{ui}$  for  $i = 1, 2, 3, \dots, n$ .

where  $X(x_1, x_2, x_3, \dots, x_n)$  is the set of independent variables with their lower and upper bounds as  $x_{li}$  and  $x_{ui}$  respectively for all the  $n$  variables.

$F(x)$  is the objective function to be optimized, since the variables are restricted with in their bounds it is considered as constraints  $g_j(X)$ .

The Sequential Unconstrained Minimization Technique (SUMT) is an indirect search method. In this method the constrained optimization problem is converted in to a series of unconstrained problem and then is solved by the Rosenbrock's method. The conversion of unconstrained is done in the following way.

At the begning of the kth iteration

$$P(X, r_k) = F(X) + r_k \sum_{j=1}^m \frac{1}{G_j(x)} \dots\dots\dots(8)$$

Where  $P(X, r_k)$  is augmented objective function. The sigma term, called the penalty term, the scalar  $r_k (r_k > 0)$  is called the penalty factor.  $G_j(x)$  is the normalized form of the all constraints  $g_j(x)$  in such a manner that these should lie between -1 and 0.0 only.

The minimization process begins with initial values of variable  $x_0$ . In order to get fast convergence, the starting value of  $r_k$  is considered so that the starting value of  $P(x, r_k)$  is twice that of  $F(X)$ . Then the augmented function  $P(x, r_k)$  is minimized with the help of a suitable unconstrained technique (the Rosenbrock's method) without any constraints to get a point say  $x_k$ . After this the new augmented function  $(X_k, r_{k+1})$  is formed with  $r_{k+1} < r_k (r_{k+1} = c * r_k)$  where  $0 < c < 1$ , and again it is minimized with  $X_{k+1}$ .

The process of unconstrained minimization is continued for decreasing sequence values of  $r_k$ , till the convergence criterion is satisfied. The convergence criterion is considered such that the process is continued for either a predetermined no of iteration or until the progress in per unit value of objective function becomes less than a small specified quantity (say e).

**Rosenbrock' Method of Rotating Coordinates [3]**

At the first stage of this method the variables are varied in sequence in both the directions parallel to their axes. When for a particular variable, there is an improvement in the value of objective function that is success, the step length corresponding to this variable is magnified by an accelerator factor  $\alpha (\alpha > 1)$ , in the search direction for the next cycle. If there is no improvement in the current value of the objective function, that is a failure, the step length of this variable is decreased by decelerating factor  $(0 < \beta < 1)$  in the reverse direction. The sequence is repeated until a success and failure are encountered for all variables at this stage.

In the second stage, the direction of search is given a rotation with respect to the original axes. Hence for computing the appropriate direction, the coordinate system is rotated in such a manner that the first axis oriented towards the locally estimated direction of a valley and all the other axes are made mutually orthogonal and normal to the first one using Gram Schmit Orthogonalization [3]. The procedure is continued for either a given no of stages or until the progress in per unit value of objective function becomes less than a specified small quantity. The best values of  $\alpha$  and  $\beta$  are taken as 3.0 and 0.5 respectively as these values are well known in the literature [3].

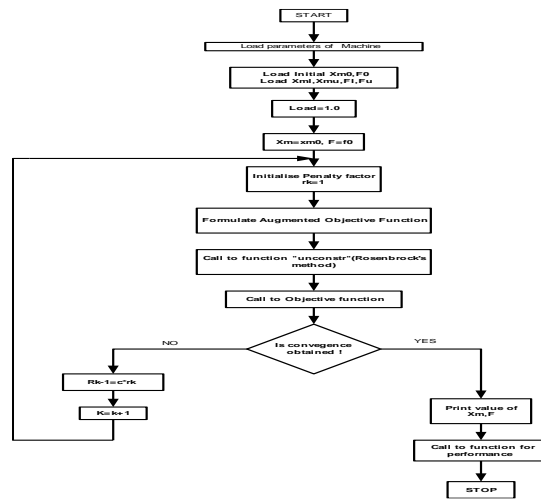


Fig-3 Flow chart of SUMT for considered SEIG case

### Identification of Air Gap Voltage

Having determined  $x_m$  and  $F$  under power generation, the next step is to identify the air gap voltage  $E_g$  corresponding to  $x_m$ . For this purpose, an  $E_g - x_m$  curve is used. This curve shows the relationship between air gap voltage  $E_g$  and magnetizing reactance  $x_m$  at rated frequency  $f_R$  which is non linear due to magnetic saturation and can be derived from the synchronous speed test [8]. Based on this curve  $E_g$  is identified by in putting the approximate relationship between  $E_g$  and  $x_m$  in to a computer program and using the method as described later in part -4.

### Calculation of Generator Characteristics

With  $E_g$  identified for  $x_m$  under given conditions, various characteristics of the generator can be calculated using the following equations:

$$I_p = E_g / ((r_1 / F) + jx_1 - j(x_{cs} / 3F^2 + ((Z_{gn} / F) - j(x_{cs} / F^2))((Z_e / 3F) + j(x_{cs} / 3F^2))) / ((Z_e / 3F) + (Z_{gn} / F) - j(2x_{cs} / 3F^2)))$$

$$I_n = -(((Z_e / 3F) + jx_{cs} / 3F^2)) / (((Z_e / 3F) + (Z_{gn} / F) - j(2x_{cs} / 3F^2))) * I_p$$

$$I = (-j(x_{cp} / F^2)) / ((R / F) + (jX) - j(x_{cp} / F^2)) * (I_p + I_n)$$

$$V = (R + jXF) * I$$

$$Po = |I|^2 * R$$

$$f = f_R * F$$

**EXPERIMENTAL SETUP & MACHINE PARAMETERS**

A standard 4-pole three-phase squirrel cage Induction Machine of 6 KW, 415 V, 12.5 A, at 50 Hz, was used as a test generator. A dc machine was coupled to this machine as prime mover.

The parameters of the induction machine used for experiment are as follows:

$$\left. \begin{aligned} r_1 &= 2.35\Omega \\ r'_2 &= 2.76\Omega \end{aligned} \right\} \text{At } f_R = 50\text{Hz}$$

$$x_1 = x'_2 = 4.47\Omega$$

These constant parameters were obtained by the dc resistance test and locked rotor test [9].

Fig-4 shows the  $E_g - x_m$  curve obtained from the synchronous test. In this analysis, to simplify the input to the computer, the operating region is divided into two parts as shown in Fig-4. The relationship between  $E_g$  and  $x_m$  was then approximated by the straight line of each region.

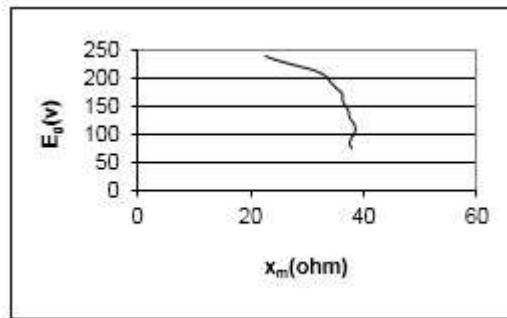


Fig-4  $E_g - x_m$  curve

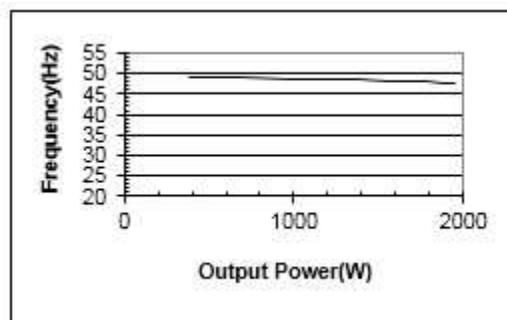


Fig-5 Frequency Vs Output power

- (1)  $E_g = 599.64 - 4.6x_m$ , for  $x_m$   $20\Omega$  to  $30\Omega$
- (2)  $E_g = 258.84 - 2.0301 x_m$ , for  $x_m$   $31\Omega$  to  $36\Omega$

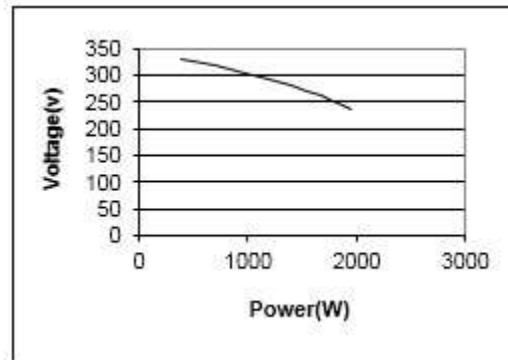


Fig-6 Voltage Versus Power

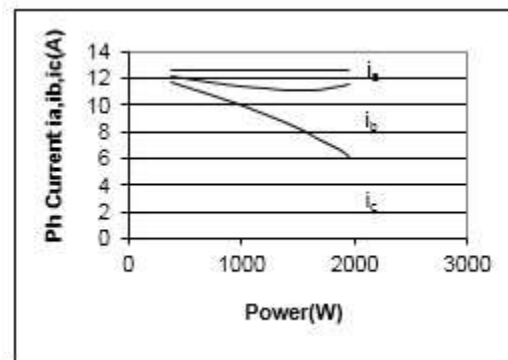


Fig-7 Phase currents  $I_u, I_v, I_w$  Vs Power

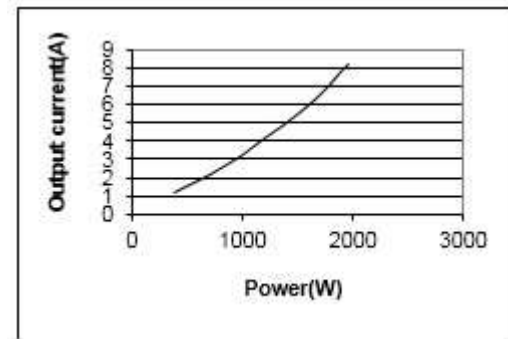


Fig-8 Output Current Vs Power

**RESULTS**

The result presented here show the case where speed of the generator has been considered at 1500 rpm. These results are produced with  $C_p = C_s = 120 \mu f$ . As the above Fig-6 clearly show, the voltage vs power characteristics of a three phase self-excited induction generator supplying single phase power, the terminal voltage of the induction generator has to decrease with the increase in the load under the fixed value of the capacitances due to the lack of capacitive VAR compensation of the SEIG and this shows the scope of a proper Induction generator controller to regulate the voltage at its output terminals. In earlier cases [2] the power output with star machine is only 30% but with the algorithm used this has improved to the value of 35%. Fig-7 shows the variation of 3-stator phase winding currents  $I_u, I_v$  and  $I_w$  w.r.t. output power of resistive load. Although the values of  $I_u, I_v$  and  $I_w$  becomes unbalanced under load conditions, there is no problem of temperature rise in stator winding while operating the



generator within its current rating. The variation of inle-phase load current has been shown by Fig-8. An almost flat curve for frequency w.r.t output power has been shown by Fig-5.

## CONCLUSION

In this paper a series parallel combination of capacitances has been used for self excitation of the Induction generator which has resulted in quite operation of generator, no vibration (since all the three phases of machine are involved in excitation process), and temperature rise with in a tolerable limit. An optimization algorithm has been used to calculate the value of  $x_m$  and generated frequency  $F$ , on which these performance characteristics have been developed. This is concluded that this study will boost the use of three phase machines for single-phase power generation cases.

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